Phase Equilibria & The Phase Rule

Dr. Moammal Qurt

Phase Equilibria & The Phase Rule Definitions

Phases: a homogeneous, physically distinct portion of a system that is separated from other portions of the system by bounding surfaces

Single phase system = Homogeneous system

Multi phase system = Heterogeneous system or mixtures

Phase Equilibrium: **Phase equilibrium** is the state of balance between two **phases** of a matter. It occurs when the transfer of matter or heat energy from one **phase** to another **phase** is equal to the rate of transfer of matter or heat energy in the reverse direction.

Phase Equilibria & The Phase Rule Definitions

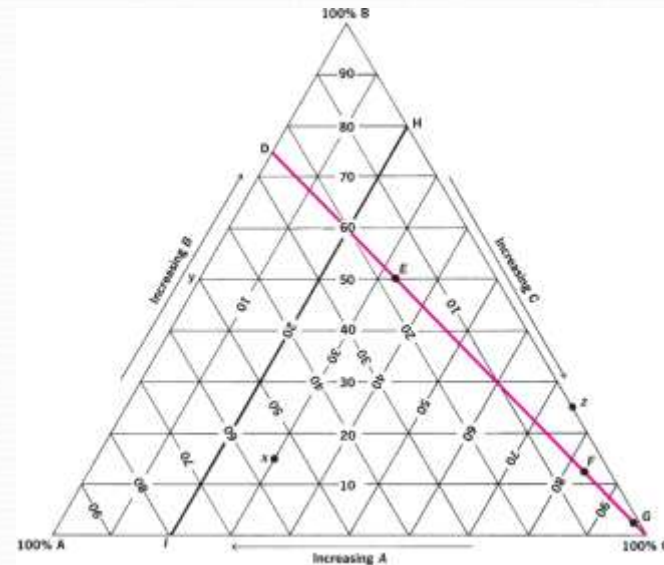Phase diagram is a graphical representation of the physical states of a substance under different conditions of temperature and pressure (variables)

A phase diagram lets you work out exactly what phases are present at any given temperature and pressure
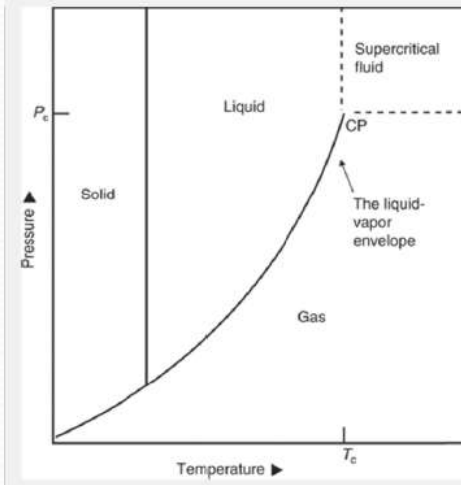
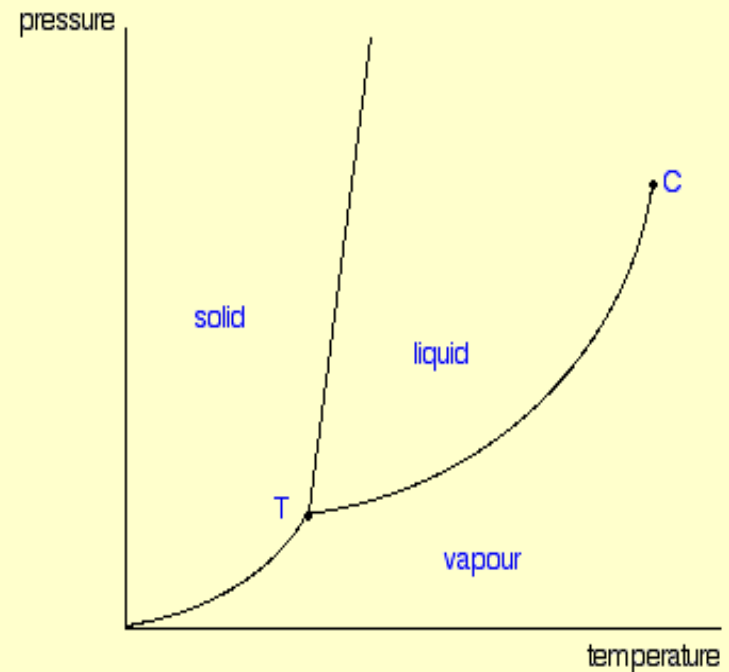Single component, 3 phases (variables are P and T

Three component Phase Variables 3 (Conc)

Systems Containing One Component

- Lets consider the following system
- Composed of one component phase diagram
- It is a 3 phase system

Solid, Liquid, gas.

- It is define by 2 varriables (P, T)

This is the phase diagram for a typical pure substance.

Systems Containing One Component

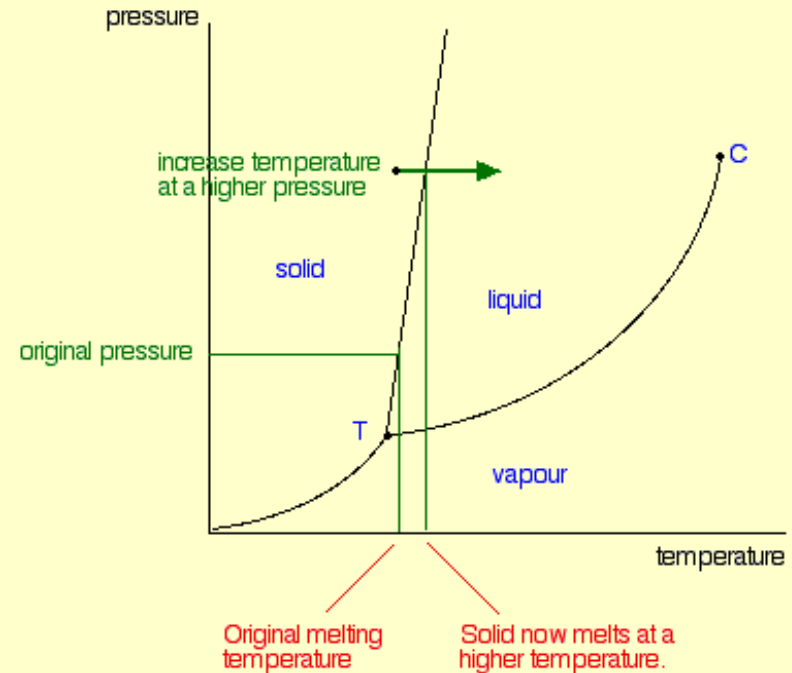- *Moving from solid to liquid by changing the temperature:*

Suppose you had a solid and increased the temperature while keeping the pressure constant - as shown in the next diagram. As the temperature increases to the point where it crosses the line, the solid will turn to liquid. In other words, it melts.


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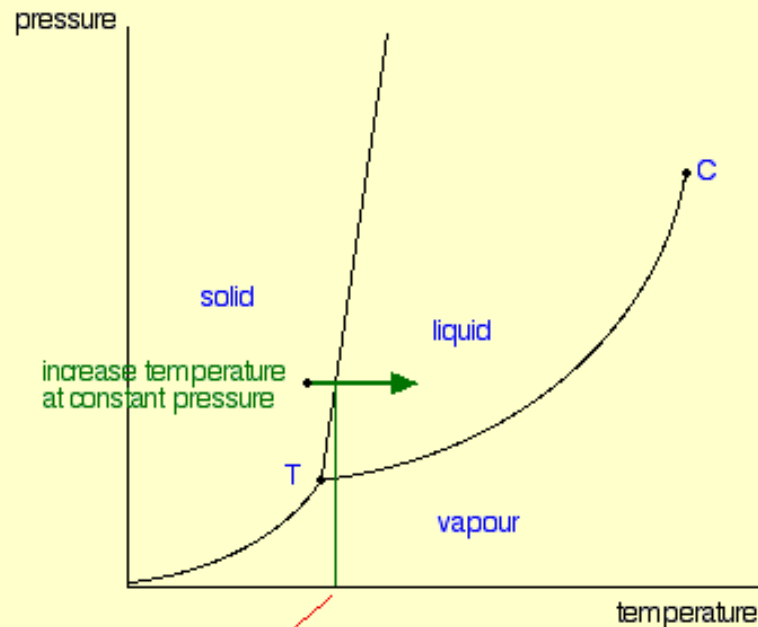
- To Identify the system in either of the single phases two variables should be determined (P, T)
- In other word to identify a point in any single phase both P,T should be identified
- That is degree of freedom is (2)
- System is called bivarient

If you repeated this at a higher fixed pressure, the melting temperature would be higher because the line between the solid and liquid areas slopes slightly forward.

pressure

increase temperature
at a higher pressure

solid

liquid

original pressure

C

T

vapour

temperature

Original melting
temperature

Solid now melts at a
higher temperature.

- The curve OA in is known as the *vaporization (or condensation) curve*.

- Any point on the curve means an equilibrium is present between liquid and vapor(2 phase curve)

- To identify the system at any point on the curve if we know one variable the other is automatically determined

- That means The degree of freedom is (1)

- The system is univariant
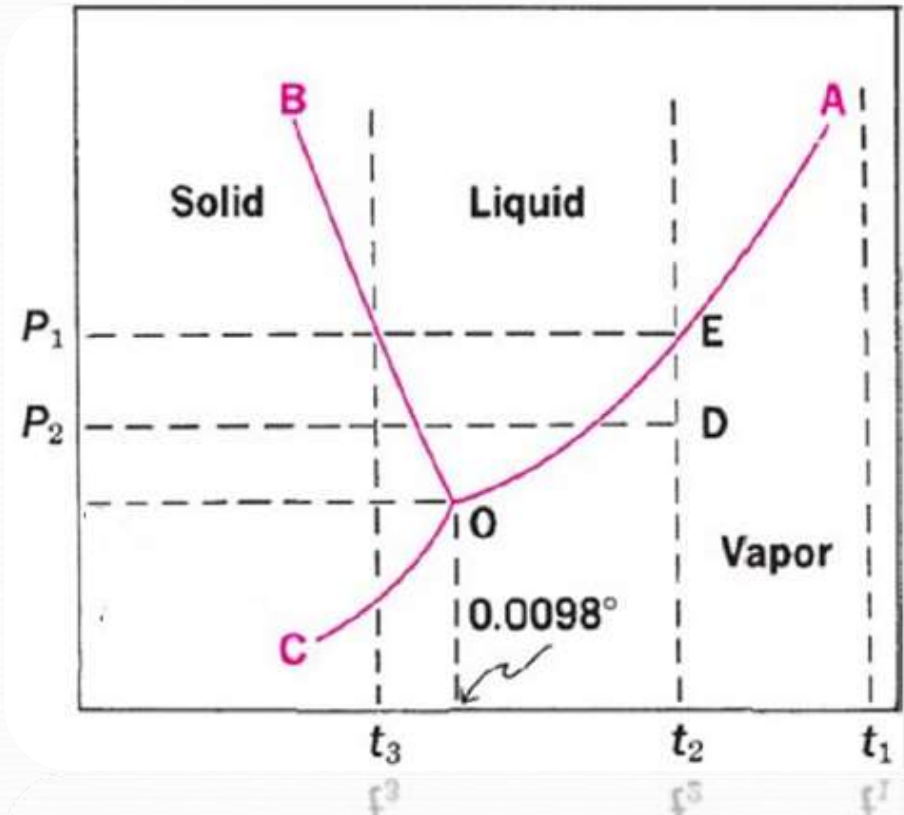
Systems Containing One Component

- **OB curve:** **Melting or freezing curve** – the curve on a phase diagram which represents the transition between liquid and solid states (2phase system)

- **OC Curve** **Sublimation (or deposition) curve** – the curve on a phase diagram which represents the transition between gaseous and solid states

- 2 phase curves

- At any point of curve the system Univariant

- Degree of freedom is 1

- I am free to chose one point the other is automatically determined

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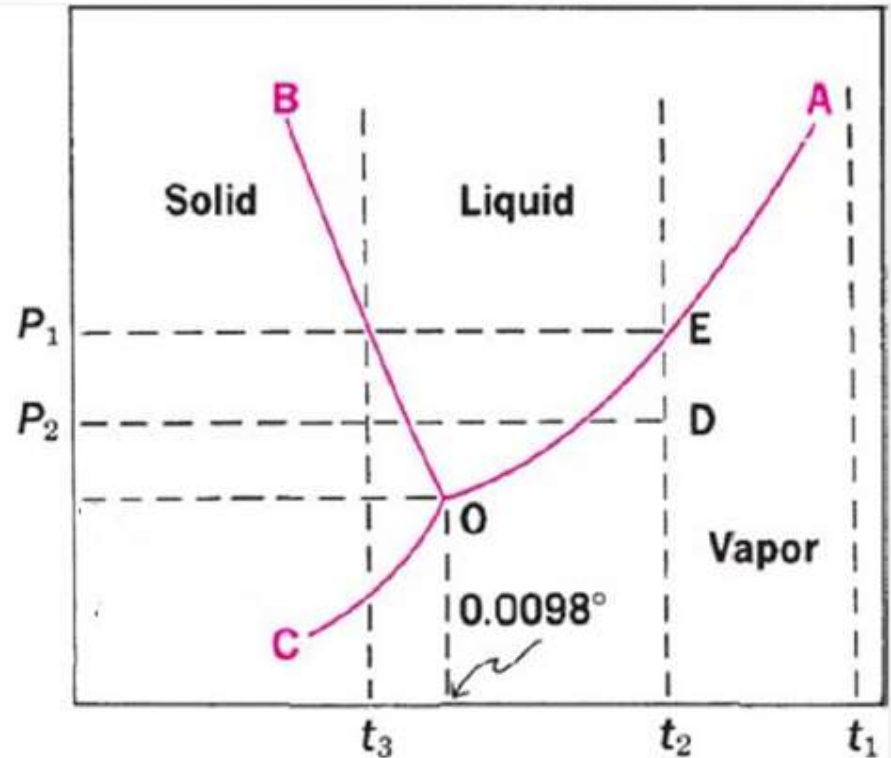
Systems Containing One Component

- The vapor curve OA Its upper limit is at the critical temperature, 374°C for water,
- its lower end terminates at 0.0098°C for water , called the *triple point*

Triple point – the point on a phase diagram at which the three states of matter: gas, liquid, and solid coexist

- a three phase system,
- P, T is predetermined to a fixed value (degree of freedom is zero)
- We are not free to choose any other P or T
- System is *invariant (can only exist at that point)*

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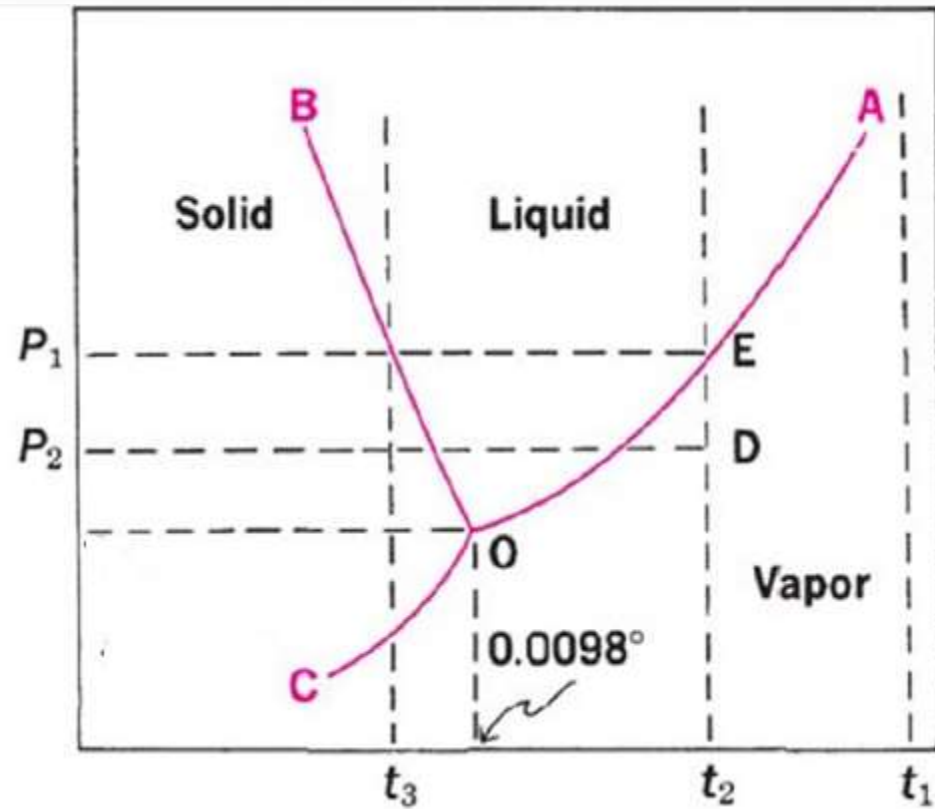
Systems Containing One Component

- P,T are called intensive variables

- **Definition of Intensive variables** : they are independent variables that do not depend on the volume or size of the phase, e.g., temperature, pressure and concentration) that can be changed without changing the phase state of the system

- Variable that Should be fixed to determined a definite point in phase diagram
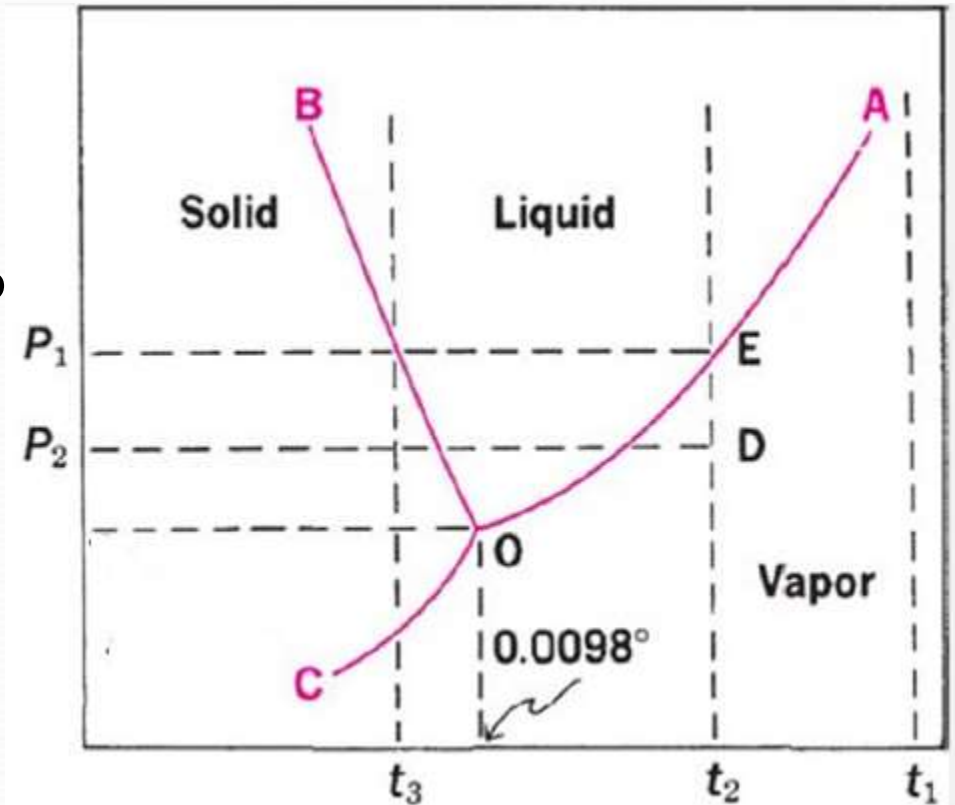
Gibb's Phase Rule

- each phase can be defined by a series of independent variables (e.g., temperature)

- to understand and defiine the state of J. Willard Gibbs formulated t he *phase rule*

- Phase Rule: which is a relationship for determining the **least number** of intensive variables (independent variables that do not depend on the volume or size of the phase, e.g., temperature, pressure, and concentration) that can be changed without changing the phase state of the system (degree of freedom)

$$F=C-P+2$$

- F: the number of degrees of freedom of the **least number** of intensive variables, e.g., temperature, pressure, density, and concentration) that can be changed without changing the phase state of the system (degree of freedom)

- C: number of system components

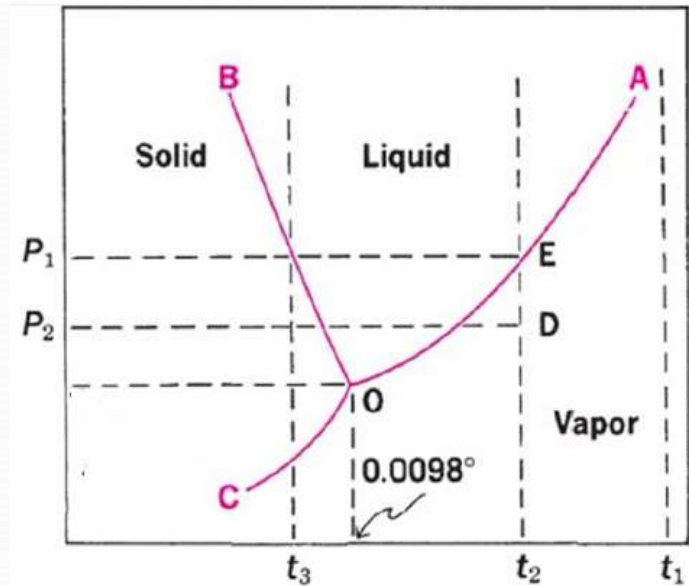- P : Number of phases at specific point in system
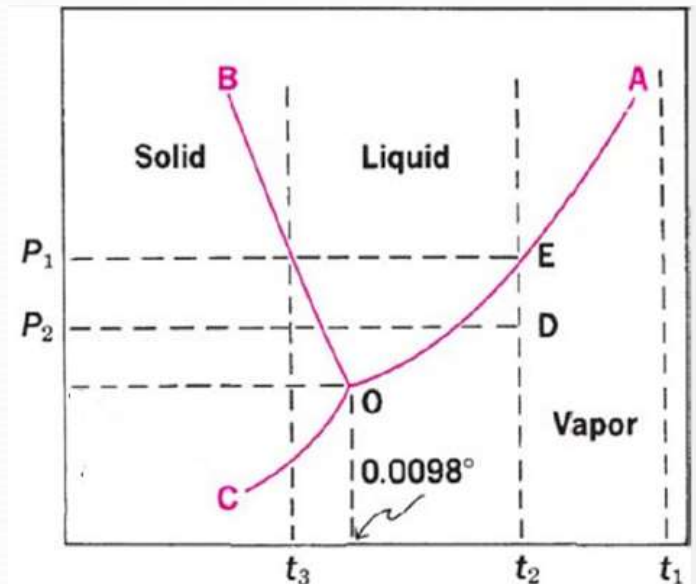
F=C-P+2

- Example at solid state of water:
- C=1 (one component water)
- P=1 (one phase solid)
- Degree of freedom (F)

F=C-P+2

F=1-1+2 ➡ F= 2

That is we are free to determine two independent variable to define the system at any point in the solid state

F=C-P+2

- Example at the melting Curve OB:

We have two phases at equilibrium

Solid ⟷ Liquid

C=1 (one component water)

- P=2 (Two phase equilbirium)
- Then Degree of freedom (F)

F=C-P+2, F=1-2+2 ⟶ F= 1

That is : we are free to determine one independent variable the other is automatically determined

F=C-P+2

- Example at the melting Triple point:

We have 3 phases at equilibrium

Solid ⟷ Liquid ⟷ Vapor
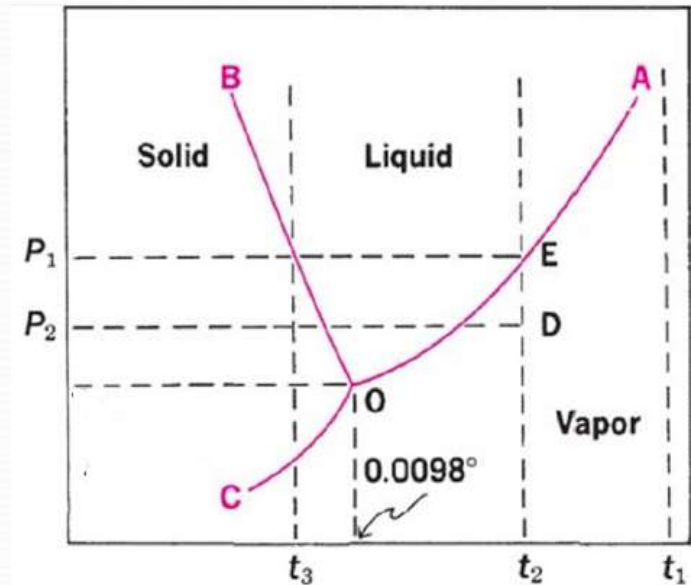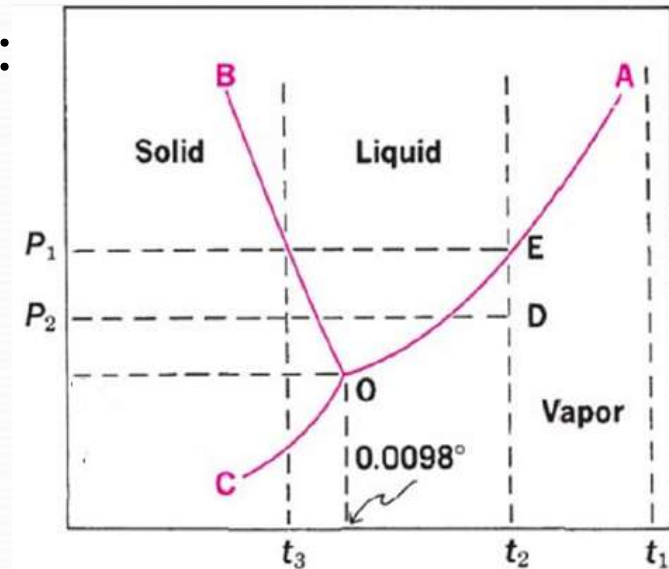
C=1 (one component water)

- P=3 (3 phase equilibrium)
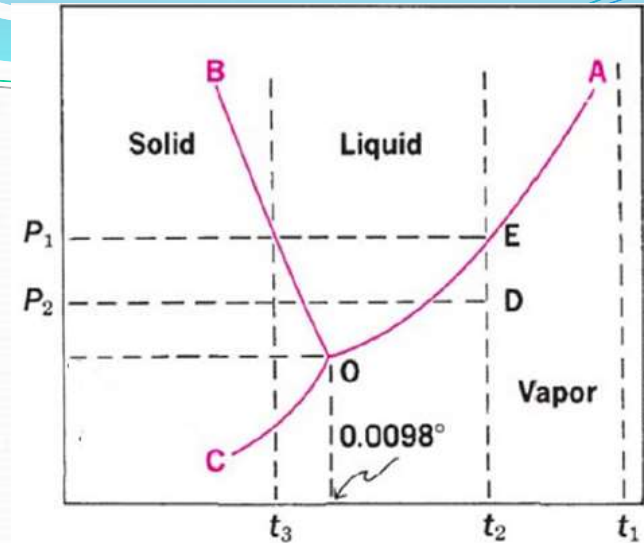- Then Degree of freedom (F)

F=C-P+2, F=1-3+2 ⟶ F= 0

That is : we are not free to determine any of the independent variable (only one value is allowed)

- Note: It must be remembered that the phase rule can only be applied to systems at equilibrium

APPLICATION OF THE PHASE RULE TO SINGLE-COMPONENT SYSTEMS*

System	Number of Phases	Degrees of Freedom	Comments
Gas, liquid, or solid	1	$F = C - P + 2$ $= 1 - 1 + 2 = 2$	System is *bivariant* ($F = 2$) and lies anywhere within the area marked vapor, liquid, or solid in Figure 2–22. We must fix two variables, e.g., P_2 and t_2, to define system D.
Gas–liquid, liquid–solid, or gas–solid	2	$F = C - P + 2$ $= 1 - 2 + 2 = 1$	System is *univariant* ($F = 1$) and lies anywhere along a *line* between two-phase regions, i.e., AO, BO, or CO in Figure 2–22. We must fix one variable, e.g., either P_1 or t_2, to define system E.
Gas–liquid–solid	3	$F = C - P + 2$ $= 1 - 3 + 2 = 0$	System is *invariant* ($F = 0$) and can lie only at the *point* of intersection of the lines bounding the three-phase regions, i.e., point O in Figure 2–22.

Key: C = number of components; P = number of phases.

Condensed system

- Assume that we have a one phase composed of two components

- The degree of freedom will be as follows

$F=C-P+2$

$F=2-1+2$ ⟶ $F=3$

Degree of freedom is 3 (three variables should be determined to define system at any phase :T, P, Conc)
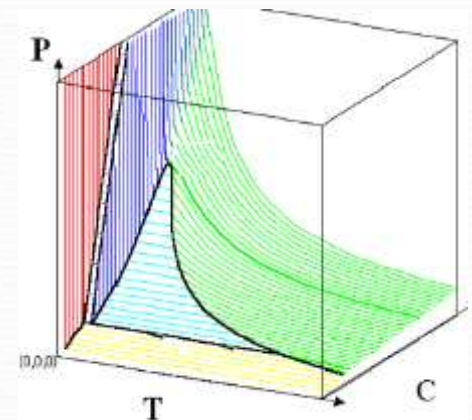
How to Plot? Go 3D but very complex

What to do ?

We can use condensed system

phase diagram

Condensed system

- Assumption made one phase composed of two components

- In pharmaceutical formulation we are mainly concerned with liquid or solid phases

- So we can discard the pressure effect and work under normal pressure 1 atm (we decrease the number of independent variables by 1

- $F=C-P+2$

- $F=2-1+2=3$ $(F-1=2)$

- We reduced the number of independent variables by 1 and now we can use a planer phase diagram to define the system

Condensed system

- They are Systems in which the pressure effect is ignored (work under 1 atm) and only solid and/or liquid phases are considered

- Advantages:

1- phase diagram can be plotted using planner phase diagram

2- in real Pharmaceutical Formulation we are concerned mainly with liquid or solid

Two-Component Systems Containing Liquid Phases

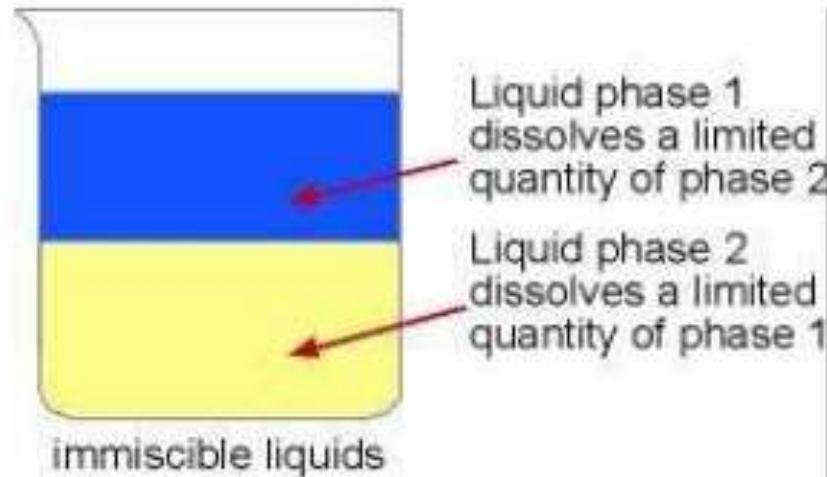- We know that ethyl alcohol and water are miscible in all proportions

Water → ← Ethanol
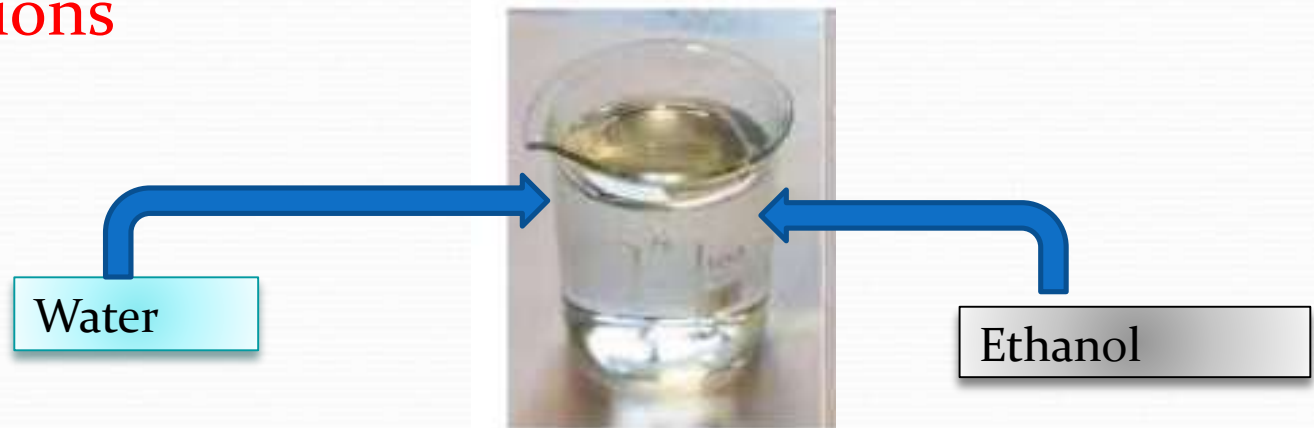
- whereas water and mercury are completely immiscible

Water   Mercury
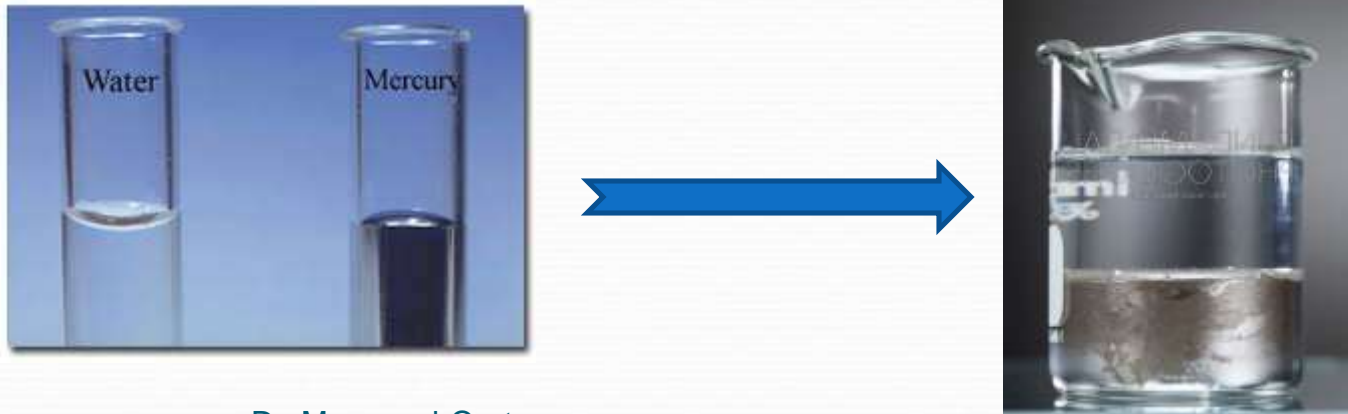
Two-Component Systems Containing Liquid Phases

- In between their some systems are Partially miscible One such system is phenol and water

Liquid phase 1 dissolves a limited quantity of phase 2

Liquid phase 2 dissolves a limited quantity of phase 1

immiscible liquids
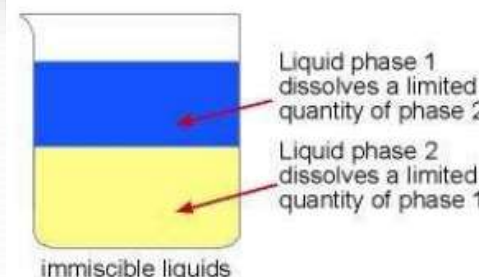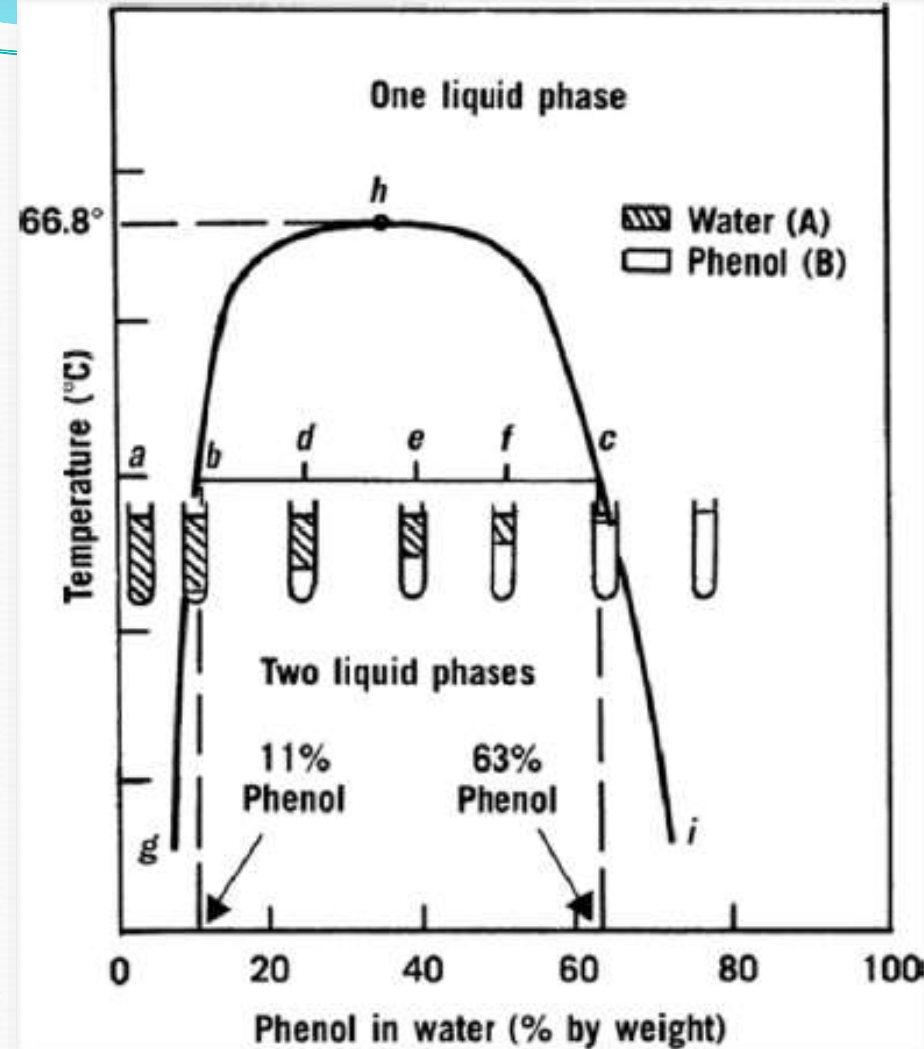
Phenol-Water

Two-Component Systems Containing Liquid Phases

- curve ***gbhci*** shows the limits of temperature and concentration within which two liquid phases exist in equilibrium (two phases inside)

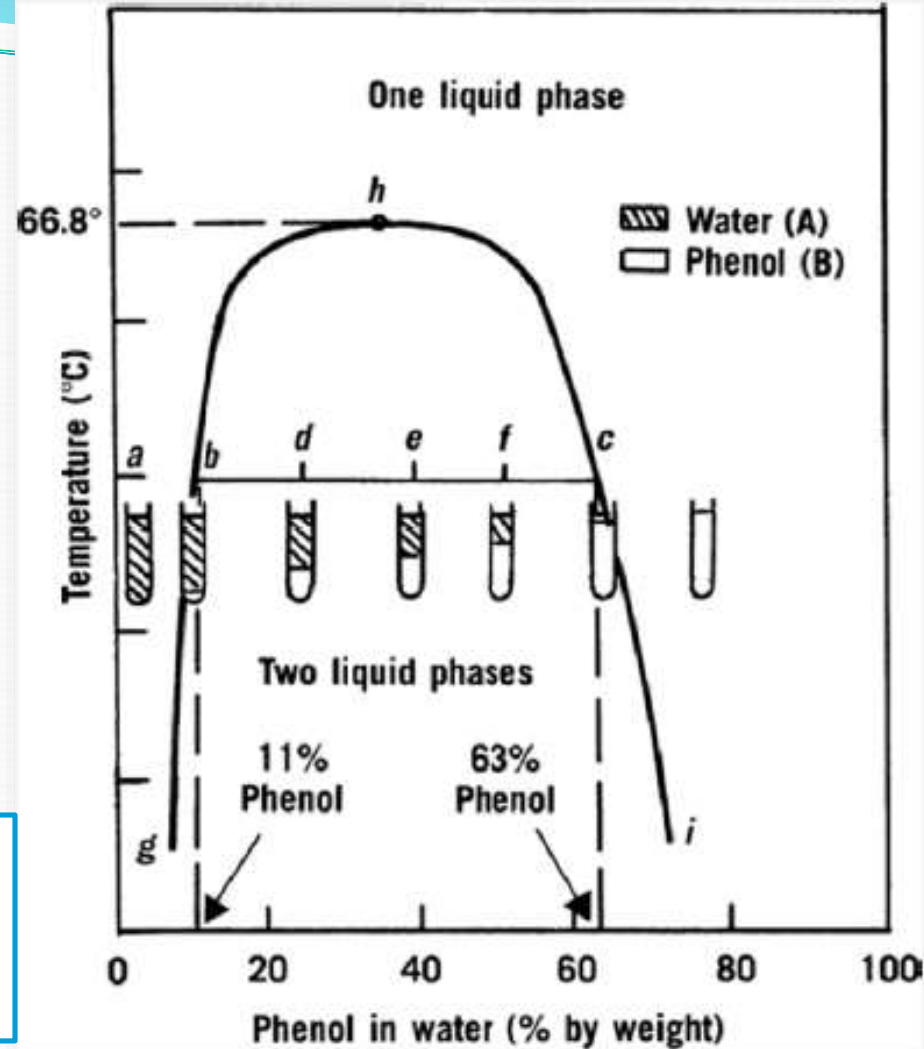- region outside this curve contains systems having one liquid phase
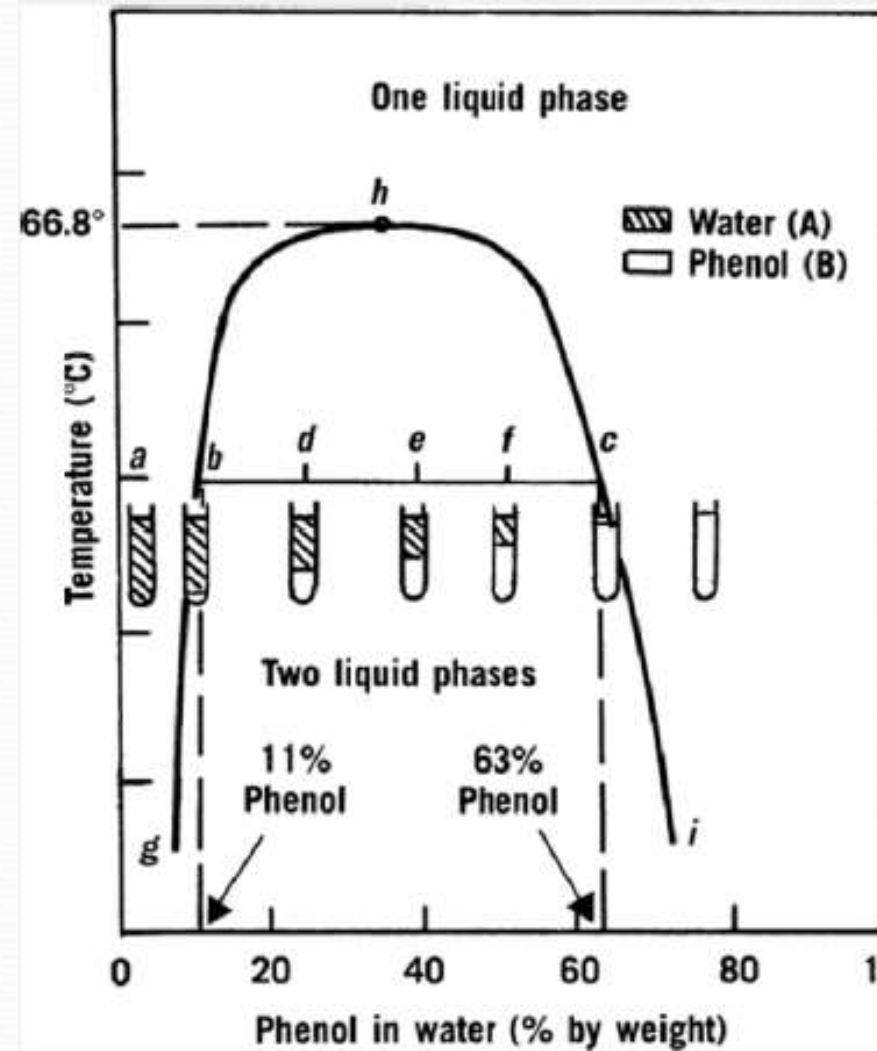
- the point **a**, equivalent to a system containing 100% water (i.e., pure water) at 50°C

- Moving from a towards b amount of phenol soluble in water is increasing until point **b** reached

- When the point **b** is reached, a small amount of a second phase appears

- At **b** Each of the phase contains phenol and water(Partially miscible)

One liquid phase

66.8°

Temperature (°C)

Water (A)
Phenol (B)

a   b   d   e   f   c

Two liquid phases

11% Phenol   63% Phenol

g   i

0   20   40   60   80   100

Phenol in water (% by weight)

Liquid phase 1 dissolves a limited quantity of phase 2

Liquid phase 2 dissolves a limited quantity of phase 1

immiscible liquids

- The concentration of phenol in water at which this occurs is 11% by weight of phenol in water

- At **b** Each of the phase contains phenol and water(Partially miscible phases)

At **b** Analysis of the second phase, which separates out on the bottom, shows it to contain 63% by weight of phenol in water
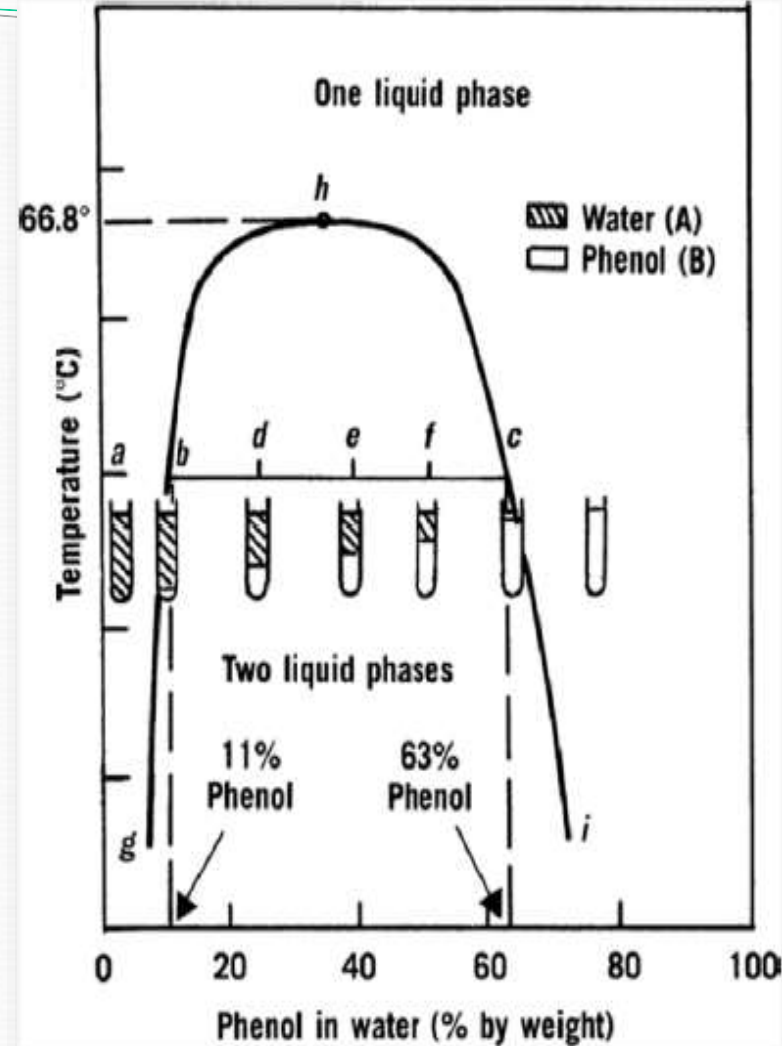

One liquid phase

XXXX Water (A)
☐ Phenol (B)

66.8°

Temperature (°C)

Two liquid phases

11% Phenol

63% Phenol

Phenol in water (% by weight)

- Poin **c** shows it to contain 63% by weight of phenol in water

- *If we move from **b** to **c**,* we form systems in which the amount of the phenol-rich phase (B) continually increases

- *If we move from **b** to **c**,* we form systems in which the amount of the Water-rich phase (A) continually decrease

- After **C** one phenol rich Phase
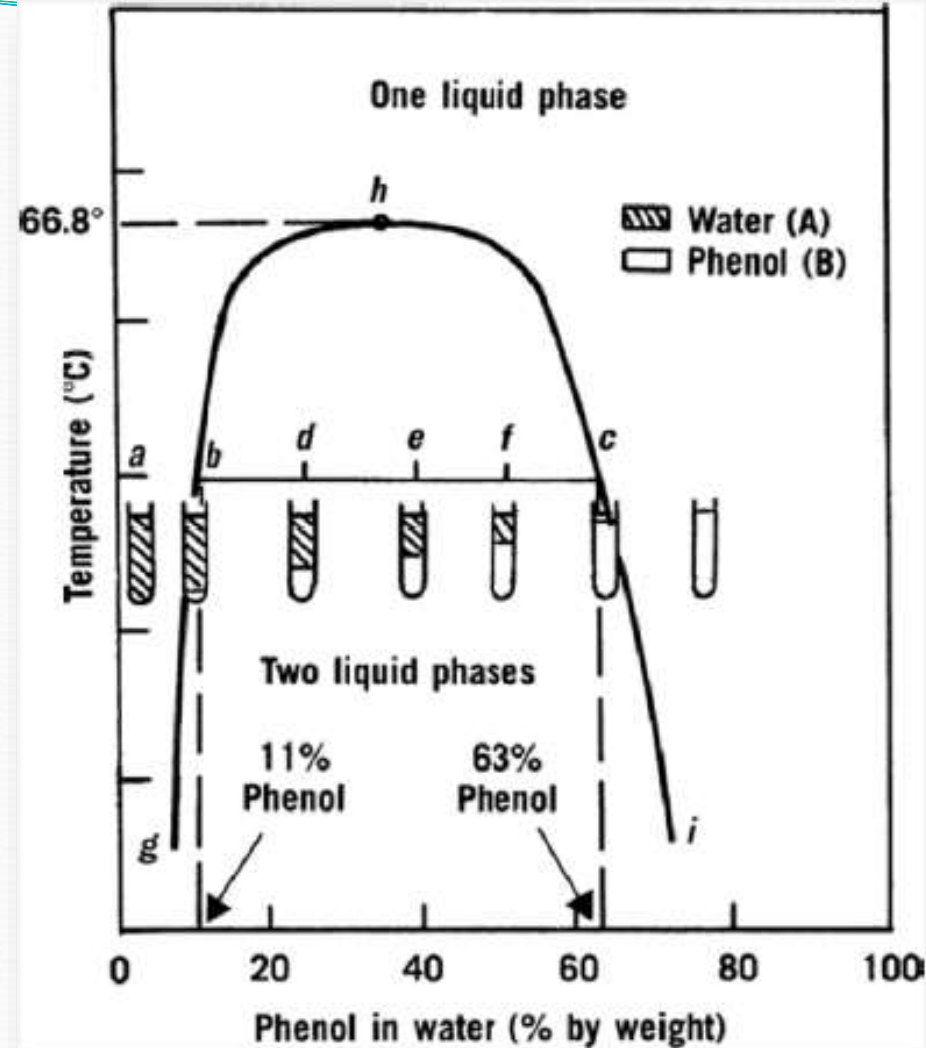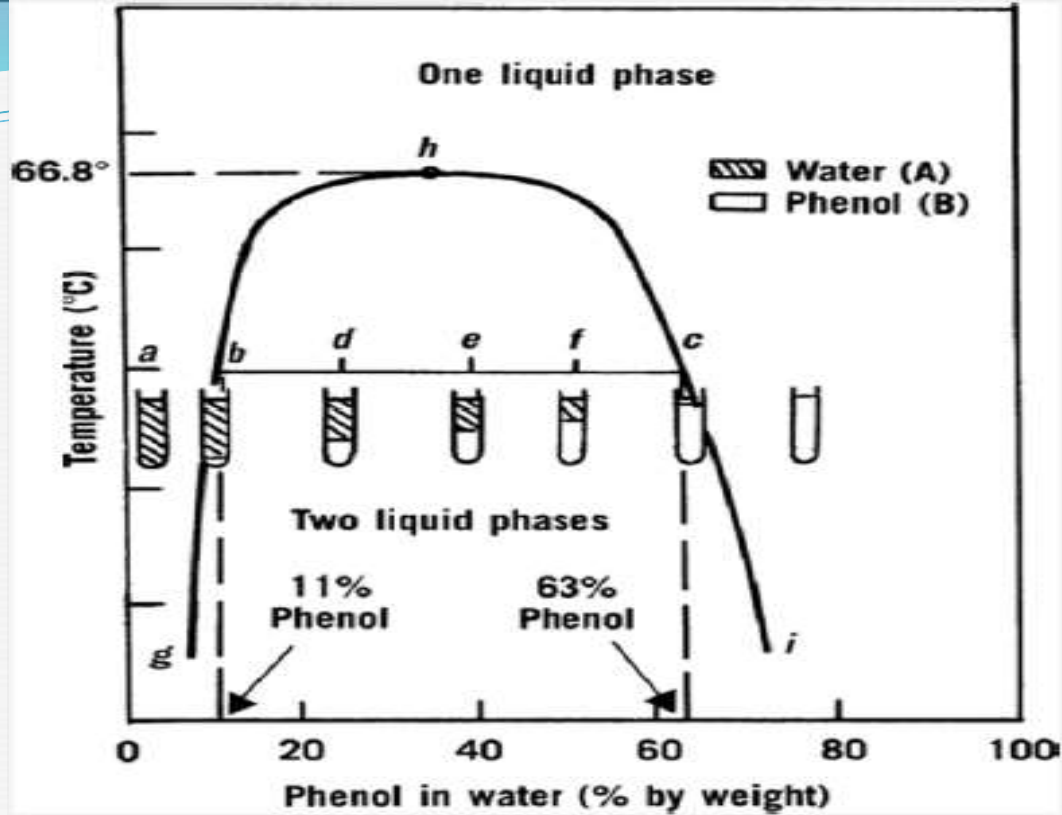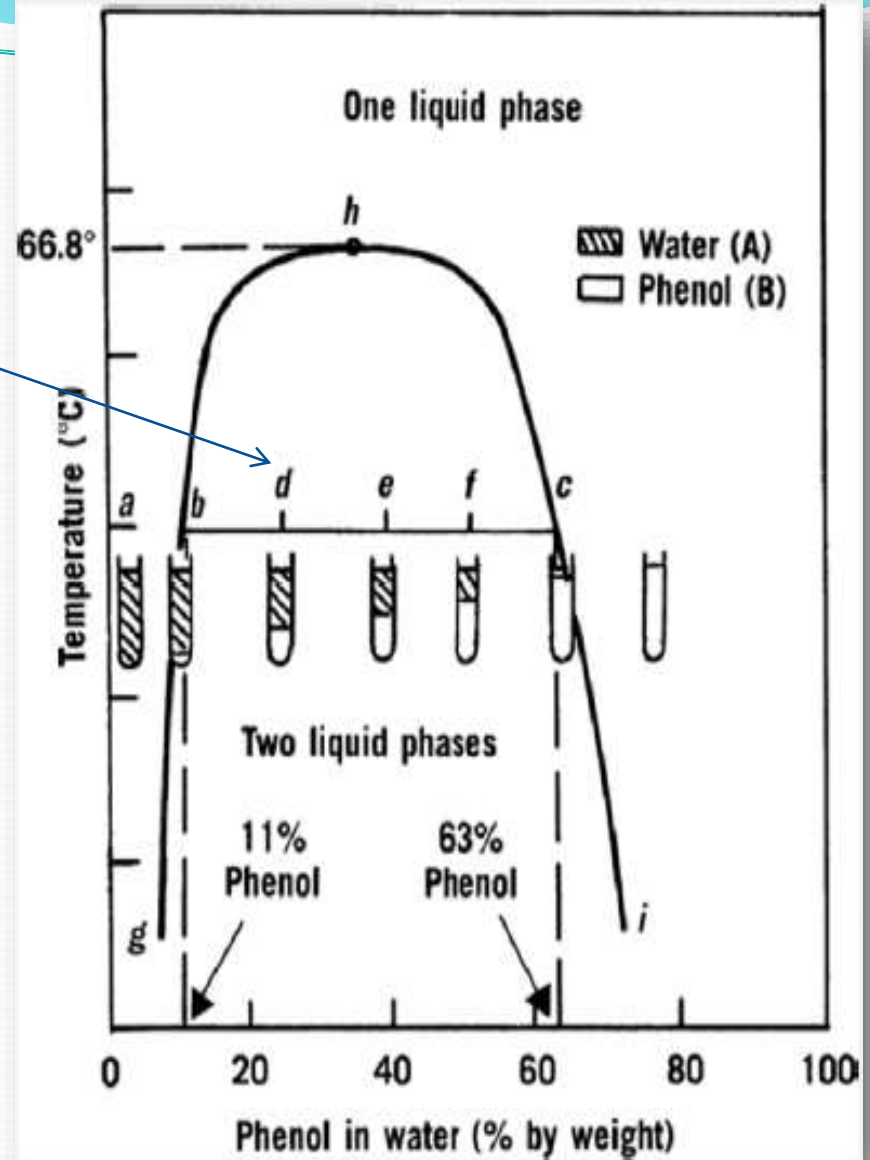
- **(point *h*):** *critical solution*, or *upper consolute, temperature*.: the maximum temperature at which the two-phase region exists (66.8°C)

- All combinations of phenol and water above this temperature are completely miscible (One Phase)

- **Tie Line:** (*bc*) drawn across the region it is al ways parallel to the base line in two-component systems (special feature)

- all systems prepared on a tie line, at equilibrium, will separate into phases of constant composition. These phases are termed *conjugate phases*

- any system represented by a point on th e line **bc** at 50°C separates to give a pair of conjugate phases whose compositions are *b* and *c*.

One liquid phase

66.8°   h

Temperature (°C)

a   b   d   e   f   c

Water (A)
Phenol (B)

Two liquid phases

11% Phenol   63% Phenol

g   i

0   20   40   60   80   100

Phenol in water (% by weight)

- if we prepare a system containing 24% by weight of phenol and 76% by weight of water (point *d*), we have two liquid phases present in the tube.

- The upper one, A, has a composition of 11% phenol in water (point *b* on the diagram), whereas the lower layer, B, contains 63% phenol (point *c* on the diagram)
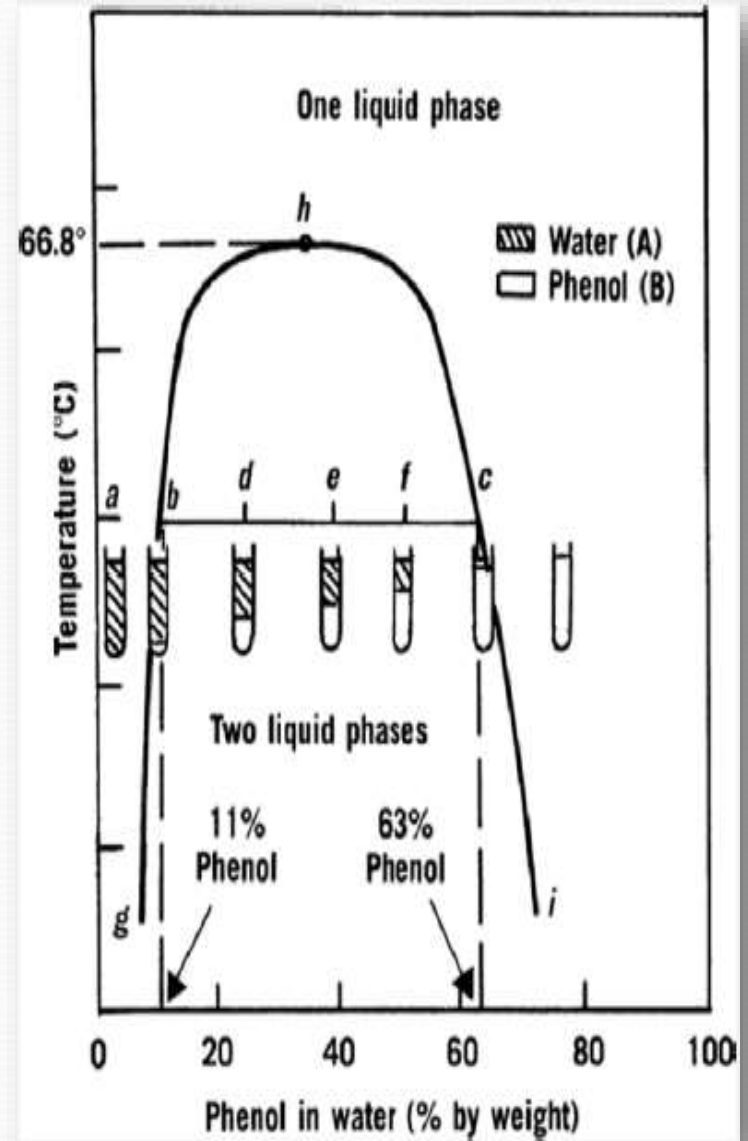

One liquid phase

Water (A)
Phenol (B)

Temperature (°C)

66.8°

h

a *b* *d* *e* *f* *c*

Two liquid phases

11% Phenol   63% Phenol

g

i

0   20   40   60   80   100

Phenol in water (% by weight)

- Phase B will lie below phase A because it is rich in phenol, and phenol has a higher density than water

- At d pint there will be more of the water-rich phase A than the phenol-rich phase B at point d.

$$\frac{\text{Weight of phase A}}{\text{Weight of phase B}} = \frac{\text{Length } dc}{\text{Length } bd}$$

- the ratio dc/bd = (63 - 24)/(24 - 11) = 39/13 = 3/1 (relative weight)
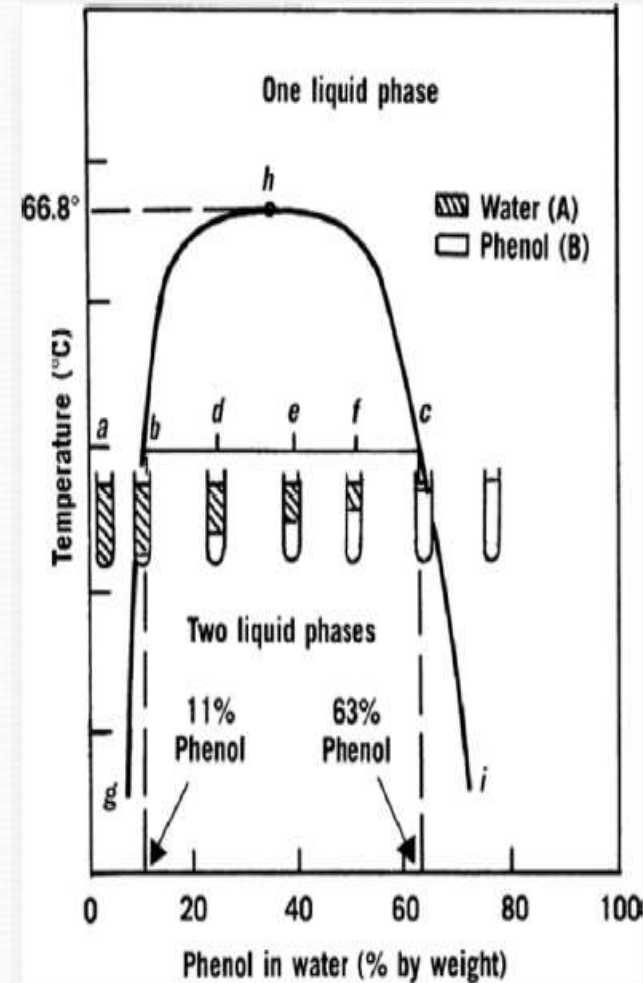
- If we prepare a system containing 50% by weight of phenol (point *f*,)
- Calculate the ratio of phase A to phase B

- The ratio of phase A to phase B is *fc*/*bf* = (63 - 50)/(50 - 11) = 13/39 =1/3

One liquid phase

66.8°

h

▧ Water (A)
▢ Phenol (B)

Temperature (°C)

a *b*   *d*   *e*   *f*   *c*

Two liquid phases

11% Phenol

63% Phenol

g

i

0   20   40   60   80   100

Phenol in water (% by weight)

- example, let us suppose we mixed 24 g of phenol with 76 g of water, warmed the mixture to 50°C, (Point *d)*

$$\frac{\text{Weight of phase A}}{\text{Weight of phase B}} = \frac{\text{Length } dc}{\text{Length } bd}$$

- the ratio *dc/bd* = (63 - 24)/(24 - 11) = 39/13 = 3/1 (relative weight) Water/Phenol
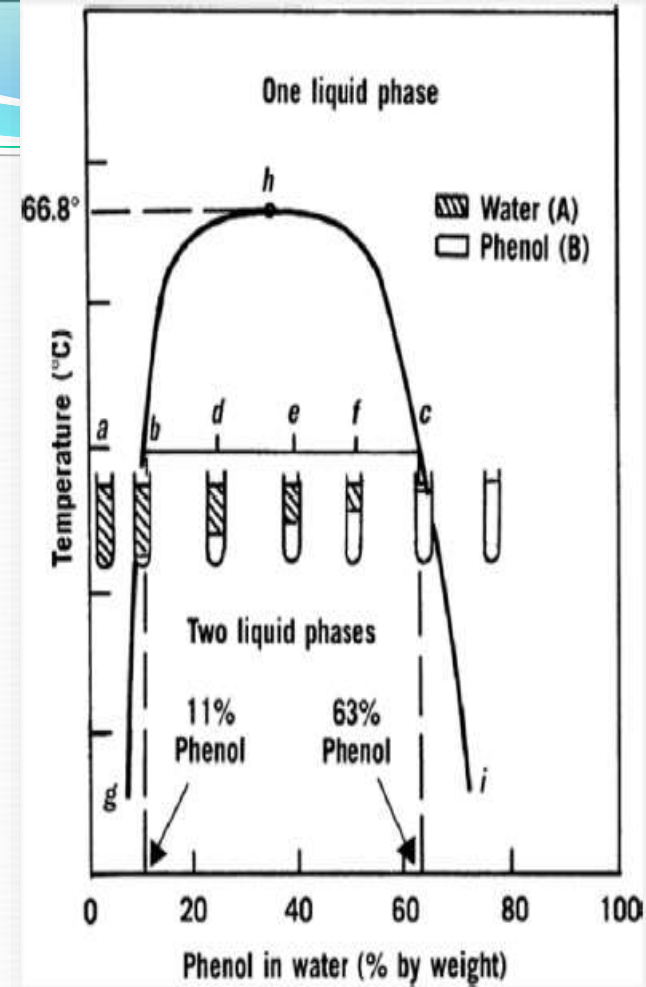
That means :

- A: Water rich phase 75 g(75%)
- B: Phenol rich phase 25g(25%)

Water rich phase on tie line contains 11% phenol

Phenol rich phase on any point on tie line have 63% phenol

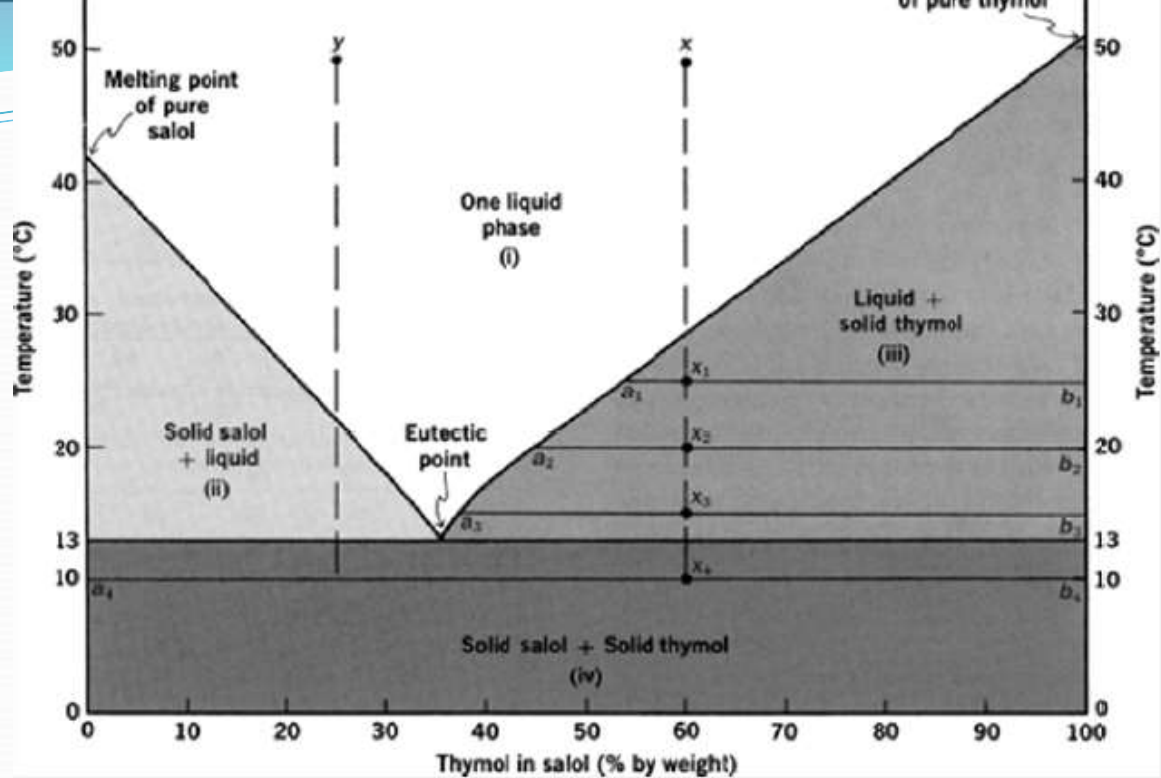Phase A therefore contains a total of (11 × 75)/100 = 8.25 g of phenol

Phase B contains a total of (63 × 25)/100 = 15.75 g of phenol

One liquid phase

Water (A)
Phenol (B)

Temperature (°C)

66.8°

Two liquid phases

11% Phenol

63% Phenol

Phenol in water (% by weight)

Two-Component Systems Containing Solid and Liquid Phases: Eutectic Mixtures


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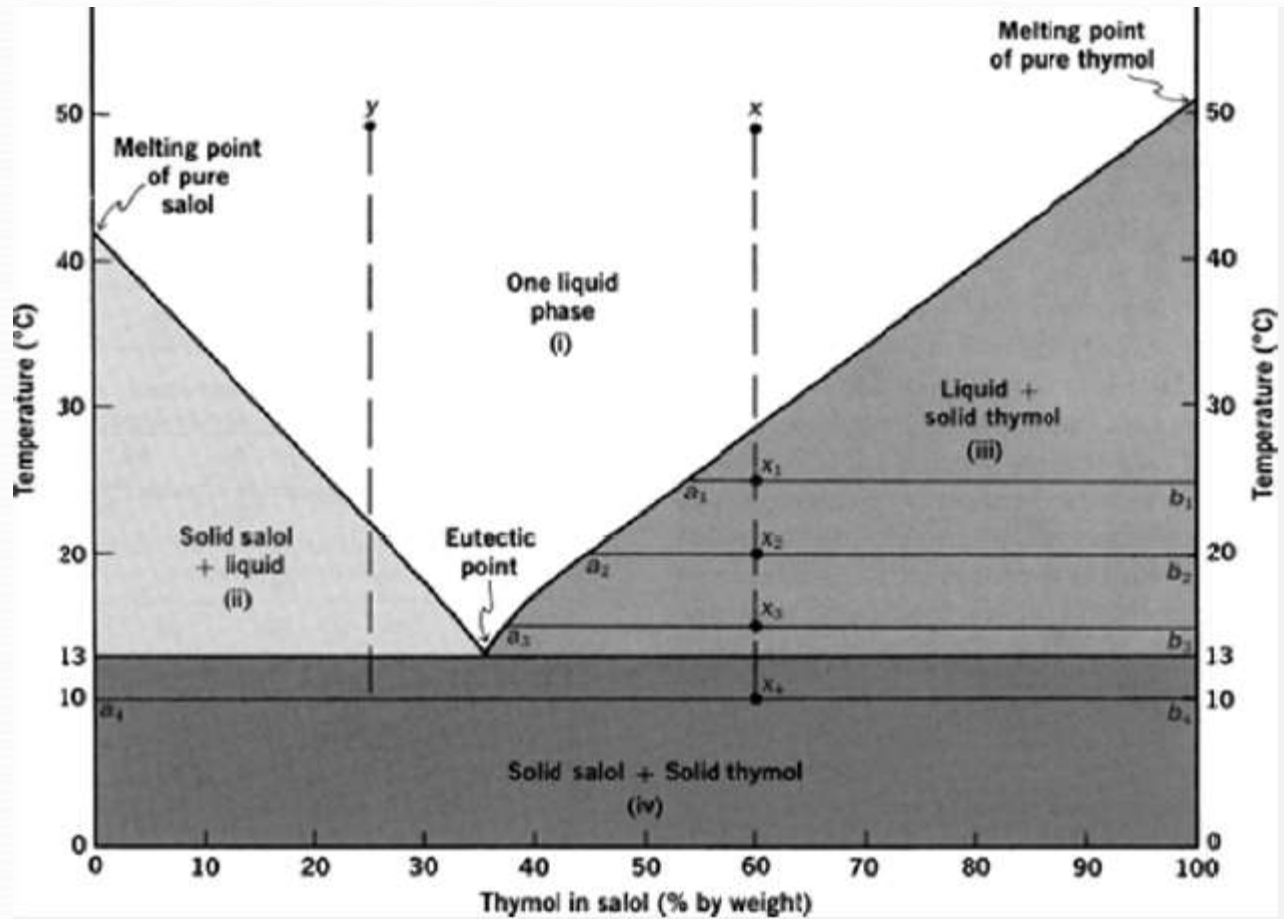
- (i) a single liquid phase

- (ii) a region containing solid salol and a conjugate liquid phase

- (iii) a region in which solid thymol is in equilibrium with a conjugate liquid phase

- (iv) a region in which both components are present as pure solid phases.

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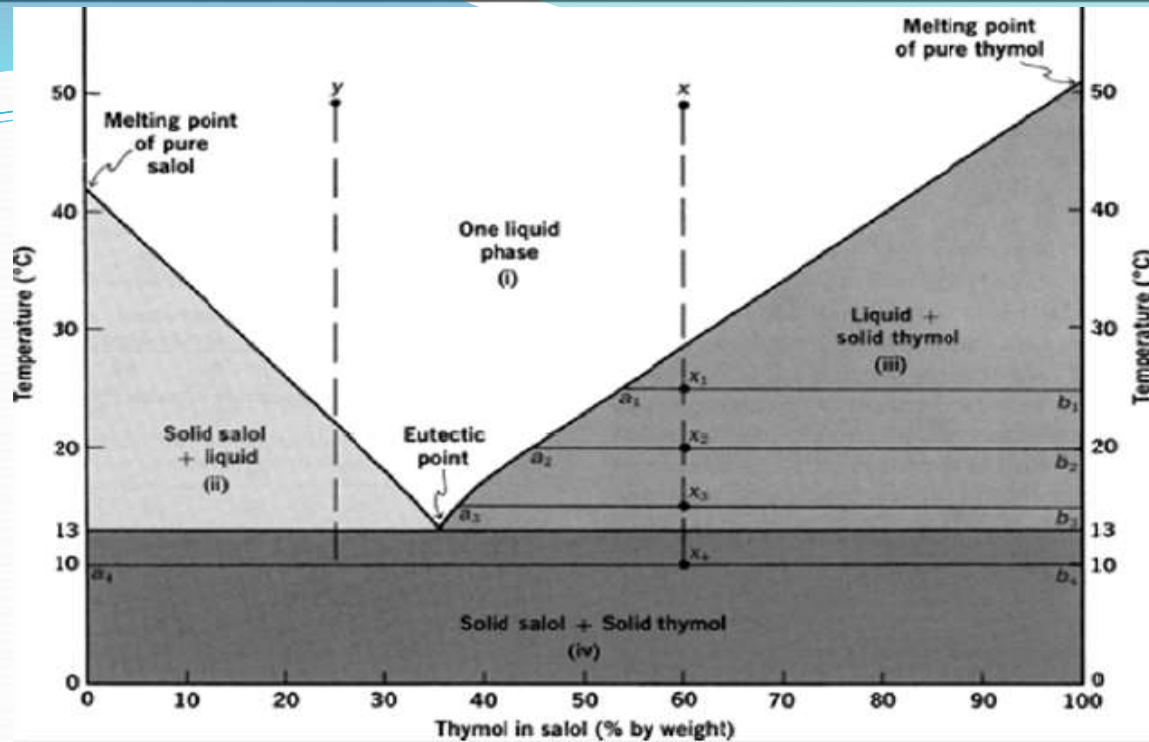
- Suppose system containing 60% by weight of thymol in salol , 50C Temp (x)"(one liquid phase.)

- Going down , At (29 C) another **solid** phase will begin to appear **(small amount of solid thymol separates)**

- (two- phase solid–liquid system)
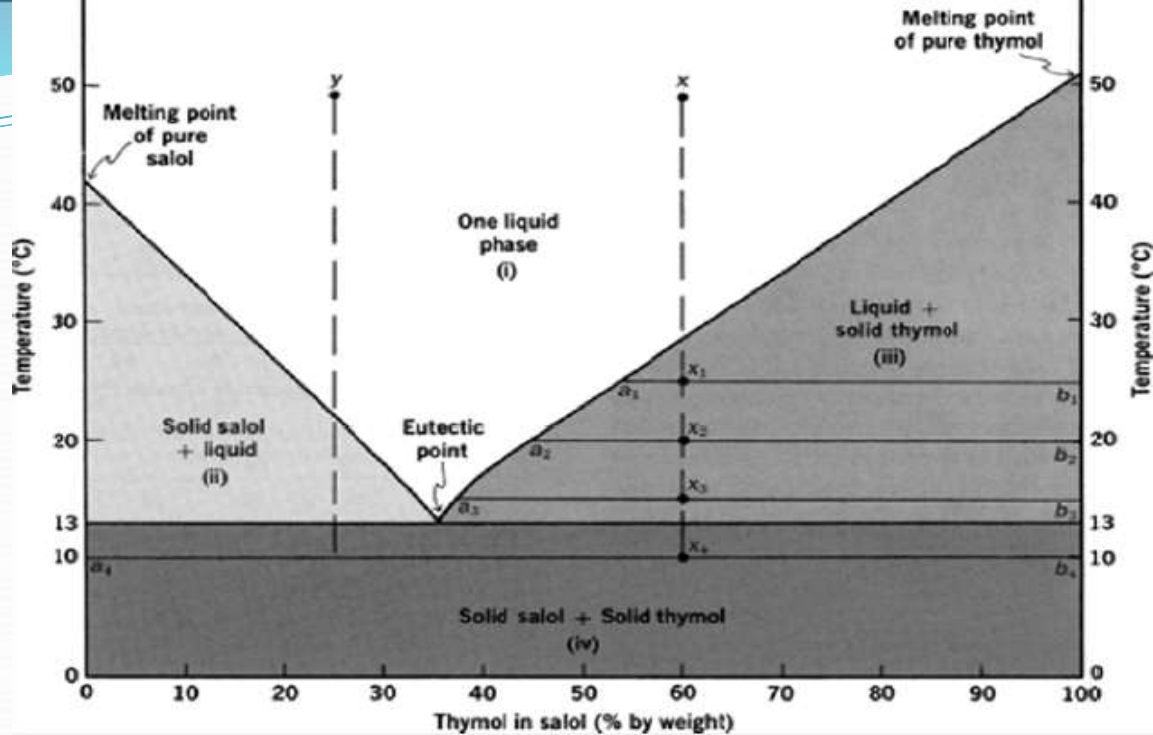
- At 25°C (room temperature), x_1 is composed of a liquid phase, a_1 (composition 53% thymol in salol), and pure solid thymol, b_1. The weight ratio of a_1 to b_1 is $(100 - 60)/(60 - 53) = 40/7$
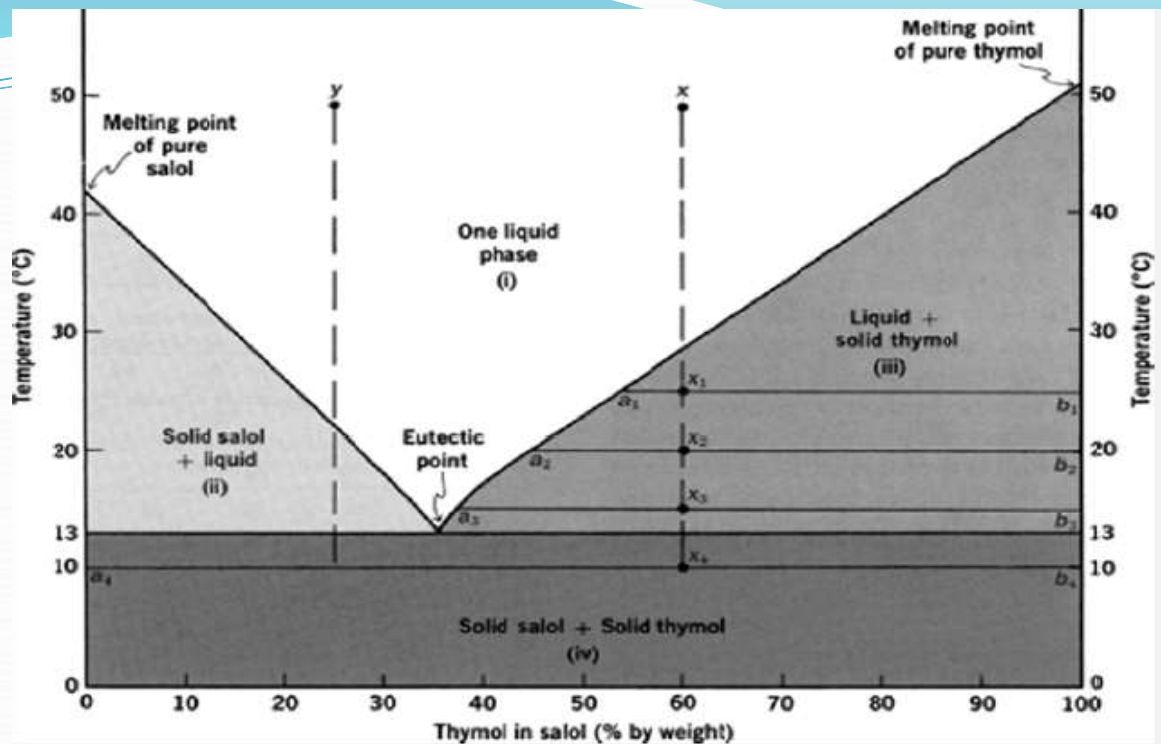
- When the temperature is reduced to 20°C (point x_2), the composition of the liquid phase is a_2 (45% by weight of thymol in s alol), whereas the solid phase is still pure thymol, b_2

- The phase ratio is a_2:b_2 = (100 - 60)/(60 - 45) = 40/ 15 = 2.67:1
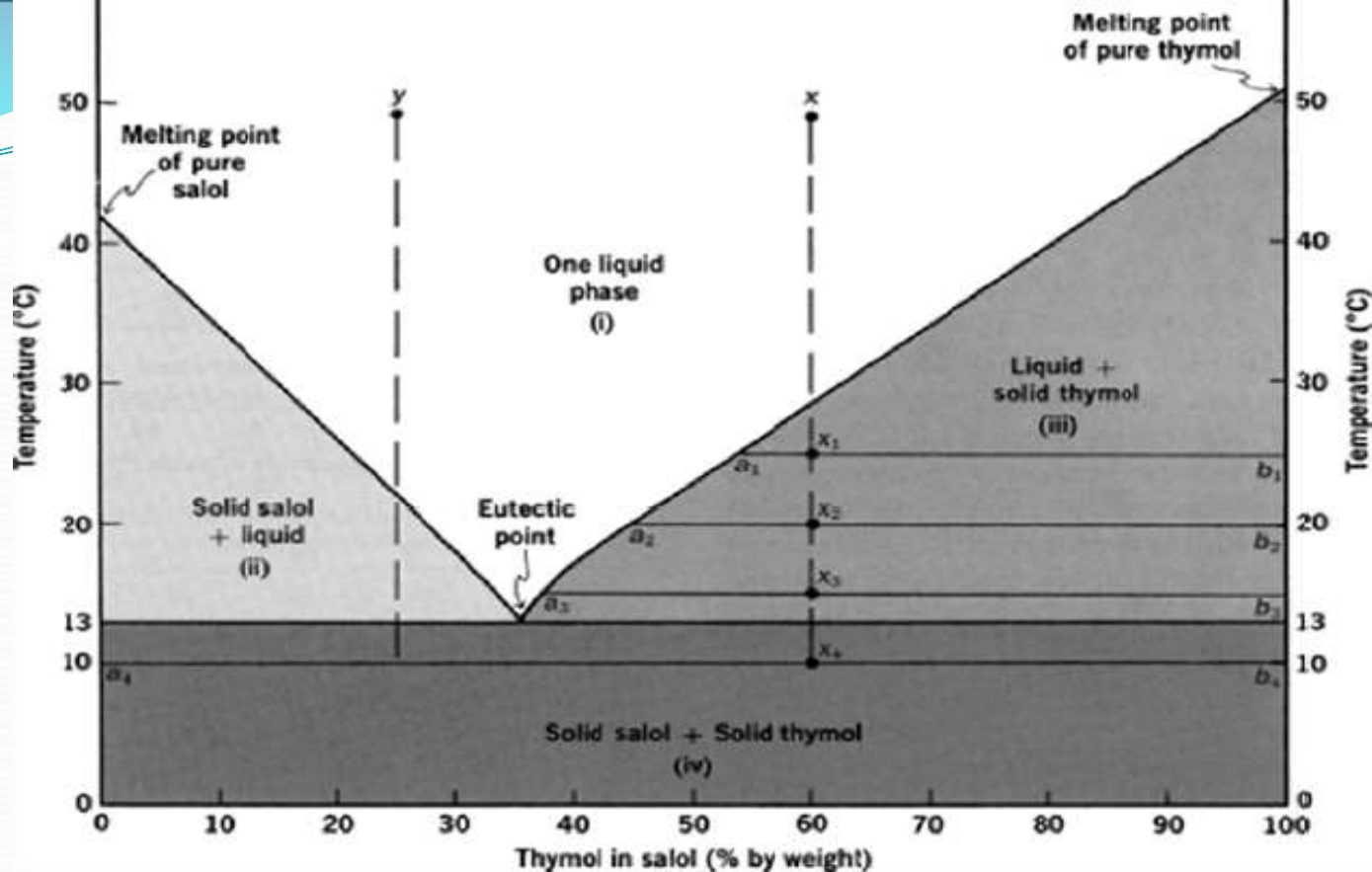
- . At15°C (point x_3), the composition of the liquid phase 37% thymol in salol (a_3) and the weight ratio of liquid phase to pure solid thymol ($a_3:b_3$) is (100 - 60)/(60 - 37) = 40/23 = 1.74:1
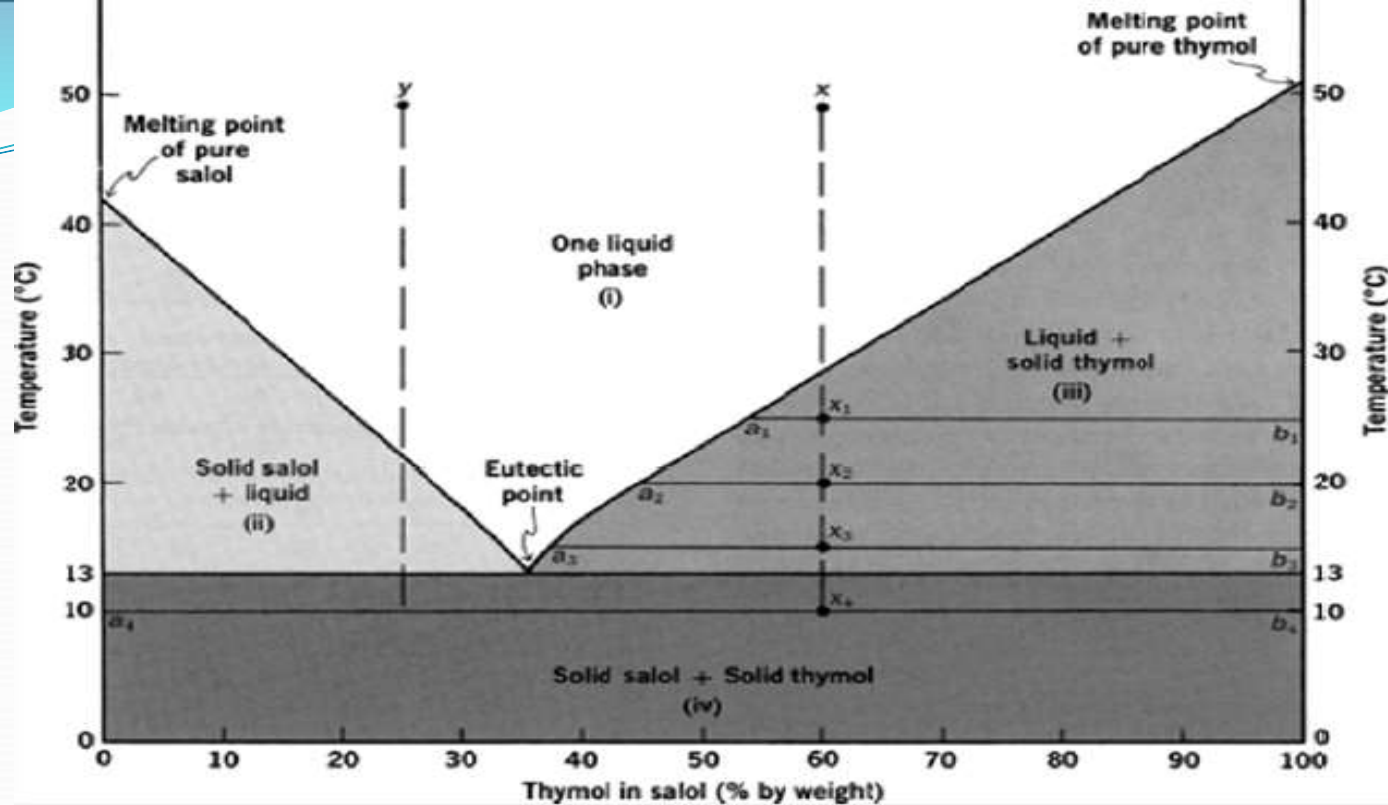
- Belo w 13°C, the liquid phase disappears
- system contains two solid phases of pure salol and pure thymol.
- Thus, at 10°C (point x_4), the system contains an equilibrium mixture of pure solid saloll (a_4) and pure solid thymol (b_4) in a weight ratio of $(100 - 60)/(60 - 0) = 40/60 = 0.67:1$.
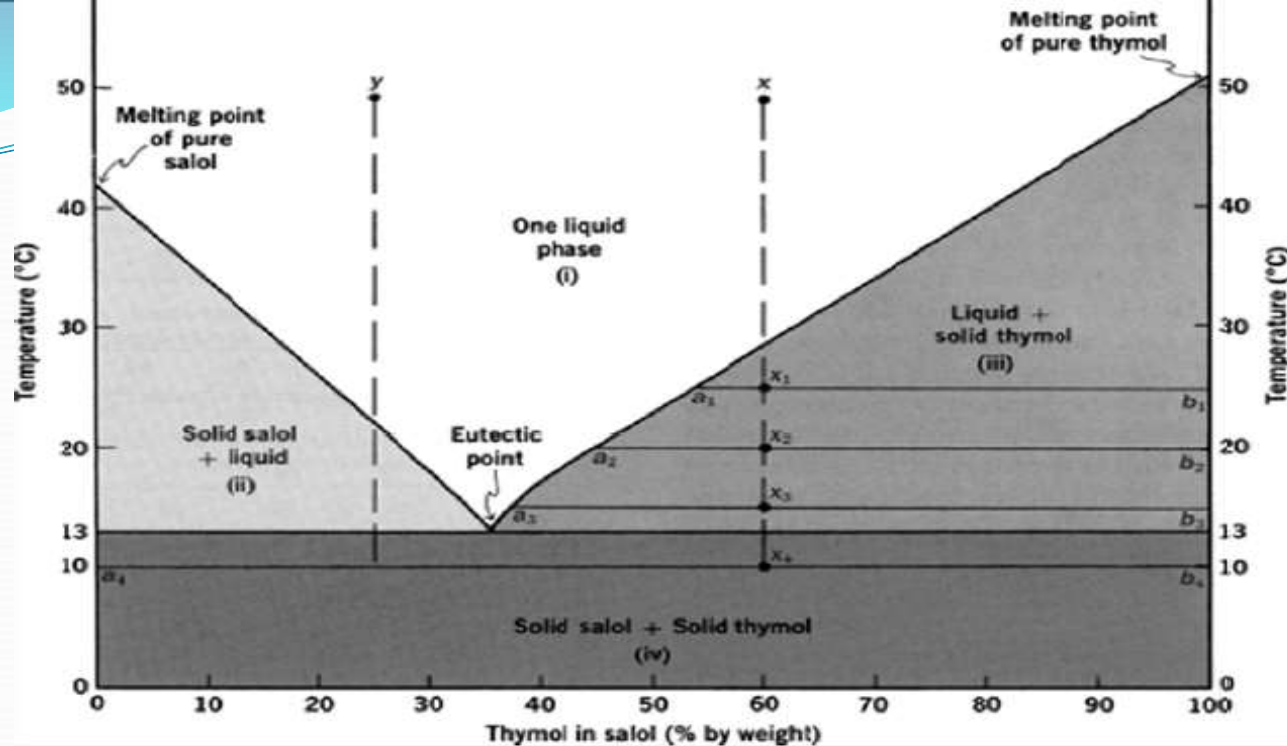
- The lowest temperature at which a liquid phase can exist in the salol–thymol system is 13°C, and this occurs in a mixture containing 34% thymol in salol. This point on the phase diagram is known as the *eutectic point*

- Eutectic point lowest melting temperature over a composition range

Eutectic system

- **Eutectic systems:** A eutectic mixture is a physical mixture of two components that do not interact to form a new chemical substance but at certain ratios inhibit each other's crystallization, resulting in a substance with a lower melting point than that of either of the components

- The melting point of a drug influences the solubility and hence the bioavailability.

Pharmaceutical Application

EMLA® (lidocaine 2.5% and prilocaine 2.5%) Cream

EMLA Cream (lidocaine 2.5% and prilocaine 2.5%) is an emulsion in which the oil phase is a eutectic mixture of lidocaine and prilocaine in a ratio of 1:1 by weight. This eutectic mixture has a melting point below room temperature and therefore both local anesthetics exist as a liquid oil rather than as crystals

Can be prepared
As emulsion

Examples

- ibuprofen with methyl nicotinate
- propranolol with fatty acids
- lidocaine with menthol